On the Stability of Application-Layer Multicast Tree

Ke Xu[1], Jiangchuan Liu[2], Lizheng Fu[3], and Chunyu Liu[4]

[1] Department of Computer Science and Technology, Tsinghua University
xuke@tsinghua.edu.cn
[2] School of Computing Science, Simon Fraser University
jcliu@cs.sfu.ca
[3] Research Institute, Bitway Network Technology Ltd. Co.
fulz@bit-way.com
[4] Department of Computer Science and Technology, Tsinghua University
liuchunyu@csnet1.cs.tsinghua.edu.cn

Abstract. A tree structure has been widely used in constructing application-layer overlays. It is known that the instability of a tree will significantly reduce the performance of the overlay. In this paper, we propose a novel stochastic model that captures the (in)stability characteristics of an application-layer multicast tree. Our model has considered various important factors related to the tree stability, and we have derived closed-form solutions to a class of typical multicast trees. Our results offer a better understanding on the (in)stability of application-layer multicast trees, and also suggest three effective enhancements to improve their stability.

Keywords: Application-Layer Multicast, Stability, Stochastic model.

1 Introduction

Multicast has a broad spectrum of applications, ranging from Internet-based TV broadcasting to large-scale multi-peer games. While IP multicast provides an efficient vehicle for multicast application, its deployment remains quite limited due to many practical and political issues. Recently, application-layer multicast (ALM) is advocated by many researchers as a flexible and readily deployable alternative.

In application-layer multicast, an end user can serve as a relay. As does in IP multicast, a tree structure is widely adopted in ALM systems. Many of them have focused on forming a high quality tree structure out of the application-layer overlay, with high throughput or short delay. However, unlike IP multicast, where the internal tree nodes are dedicated routers, an overlay node is autonomous, which can join or leave the overlay at will, or even crash without notification. The (in)stability thus becomes a key problem in an ALM tree, which can significantly impact the user experience.

In this paper we present an initiative discuss on the impact of dynamic user behaviors to the stability of ALM trees. We assume that, when a parent node departs from the multicast tree, fails, or suffers from congestion, and then all its descendents will be affected. This assumption generally holds for existing tree-based ALM protocols if repairing algorithms are ignored. We then propose a novel stochastic

A. Levi et al. (Eds.): ISCIS 2006, LNCS 4263, pp. 401 – 412, 2006.

model that captures the (in)stability characteristics of an ALM tree. Our model has considered various important factors related to the tree stability, and we have derived closed-form solutions to a class of typical multicast trees. Our results offer a better understanding on the (in)stability of ALM trees, and also suggest three effective enhancements to improve their the stability.

The rest of this paper is organized as follows. Section 2 presents the related work. The models and definition of stability are discussed in Section 3. In Section 4, we analyze the stability for different multicast trees, and we further analyze the impact of stable nodes in Section 5. Finally, Section 6 concludes the paper.

2 Related Work

Application layer research has been a hotspot in today's network research. Due to the limit of network layer multicast, shift this function to application layer is a kind of solving method. Up to date, there has been considerable research in this field. In this section, we will introduce some related work of application layer multicast.

2.1 Application Layer Multicast

ALM up shifts the network layer functions that used to be realized by routers in IP multicast to application layer nodes. As such, there is no fundamental change needed in the network infrastructure. The main target of an application layer multicast protocol is to efficiently deliver data to all the end users, and we can classify them into two: unstructured ALM and structured ALM.

An unstructured application layer multicast scheme does not maintain an explicitly structure. A node can obtain data from one or more neighboring nodes through flooding, random searching, etc. In Peercast [1], there is only one neighboring node, where as in Coolstreaming [2], there are 4 neighboring nodes, which potentially enhances robustness.

The unstructured ALM is relatively simple to implement. However, they often suffer from the low efficiency without an explicit structure. On the contrary, structured application layer multicast scheme preserves an explicit structure among the overlay nodes. They maintain a control topology and a data topology. The group members in the control topology periodically exchange update messages to identify each other and recover from node failures. The data topology is a subset of control topology, and is for identifying the data routes used in multicast forwarding. In existing protocols, a tree topology is widely used; examples include Yoid [5] and Overcast [6].

2.2 Stability of Multicast Tree

Since an overlay node can join or leave at will, or even crash without notification, (in)stability becomes a great challenging in application-layer multicast. We are aware that there are several previous studies on the stability of a tree topology or multicast. The stability of a shortest path tree is defined in [8]. Zhang et al. [9] proposed a link-dependent model and the dependent-degree model for modeling congestion bottlenecks in a tree. They also defined a real-valued dependency-degree factor to quantify all possible degrees of dependency between the random variables in the Markov chain's one-step transition probabilities. In [7, 10], we have applied stochastic model analysis to the stability of layered multicast congestion control. Our work is motivated by these studies, yet we focus on the stability issues specifically in application-layer multicast.

3 Model and Definitions

In order to evaluate and quantify application layer multicast tree stability, we first give a definition of stability:

Definition 1: Assume that the total node number of multicast tree T is $N(T)$, $\Delta(T)$ represents the number of nodes affected by a malfunctioned (leave, fail, or congested) node, and $E(\Delta(T))$ represents the expected value of $\Delta(T)$. We define the stability factor of multicast tree as $S(T)=1-E(\Delta(T))/N(T)$. Intuitively, the higher the stability factor of the multicast tree T, the more stable it is.

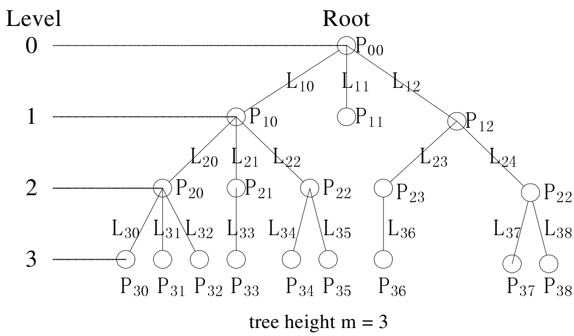

Fig. 1. An application layer multicast tree model

There are many factors that have impact on the stability of an application layer multicast tree: the probability of nodal departure, the nodal service capacity, the topological structure of the multicast tree, the height of the multicast tree, etc. In order to investigate the impact these factors, we now give a multicast tree analytical model which labels stochastically the irrelevances.

Definition 2: A random irrelevance-marked application layer multicast tree is an application layer multicast tree with labels being independent stochastic variables. The height of the tree is *m* and the node set which composes the tree is *P*. It meets these four conditions:

1: All the nodes of set P are marked in the multicast tree T, and $P=\{P_{00},P_{10},\cdots,P_{1j_1},P_{20},\cdots,P_{2,j_2},\cdots,P_{m,0},\cdots,P_{m,j_m}\}$, as is shown in Fig.1. Here,

P_{ij} represents node j (from the left) on layer i, link L_{ij} links the adjoining upstream nodes of nodes P_{ij} and P_{ij} in multicast tree T.

2: In order to identify the relation between two nodes in a multicast tree, we define some simple operations:

1) Adjoining links: we use $NU(T, P_{ij}, P_{i+1,l})$ to represent adjoining links, where $P_{i,j}$ and $P_{i+1,l}$ represent respectively a node on layer i and layer i+1 in the multicast tree. $NU(T, P_{ij}, P_{i+1,l})$ meets following expression:

$$NU(T, P_{ij}, P_{i+1,l}) = \begin{cases} 1, \textit{if } P_{i+1,l} \textit{ is the neighbor upstream node of } P_{ij} \\ 0, \textit{else} \end{cases}$$

2) upstream link set: We express the upstream link set as $NUL(T, P_{ij})$, which denotes a set $L^{'}$ for all the links in the route from root node P_{00} to node P_{ij}.

3) upstream path node set: We express the upstream path node set as $NUP(T, P_{ij})$, which denotes a set $P^{'}$ for all the nodes in the route from root node P_{00} to node P_{ij} .It is worth noting that root node P_{00} always belongs to set $P^{'}$, and $NUP(T, P_{00}) = \{P_{00}\}$.

3: Assume that the stochastic variable X_{ij} represents the mark state of node P_{ij}, and $X_{ij} \in \{0,1\}$. We have: $P_r\{X_{ij} = x_{ij}\} = \begin{cases} p_{ij}, x_{ij} = 1; \\ 1 - p_{ij}, x_{ij} = 0; \end{cases}$. p_{ij} represents the probability of node P_{ij} 's departure from the application layer multicast tree, and $0 < p_{ij} < 1$.

4: The mark probabilities of nodes are independent to each other.

With reference to definition 2, we can give the definition of a bottleneck node.

Definition 3: For an irrelevance-marked application layer multicast tree (height =m), if a nonleaf node P_{ij} departs from the multicast tree but no other node in the route from node P_{ij} to root node P_{00} departs from the multicast tree, then this node P_{ij} is called a bottleneck node of the multicast tree.

The following theorem gives the stability factor of an ALM tree.

Theorem 1: For a relevance-marked application layer multicast tree T (height = $m(m \geq 2)$) as given by definition 2, $SubT(T, P_{ij})$ defines a subtree in tree T, whose root node is node P_{ij}. When $P_{ij} = P_{00}$, $SubT(T, P_{00}) = T$. If we use $\Delta(T)$ to represent the number of nodes departed from the tree T due to the existence of the bottleneck node, then the expected values of $\Delta(T)$, $E(\Delta(T))$ and stability factors $S(T)$ are:

$$E(\Delta(T)) = \sum_{i=1}^{m} \sum_{all\ j} \Psi_d(T, P_{ij}) \cdot N(SubT(T, P_{ij})) \qquad (1)$$

$$S(T) = 1 - E(\Delta(T))/N(T) \ . \qquad (2)$$

P_{ij} is any node but P_{00} in multicast tree T, $1 \leq i \leq m$.

4 Stability Analysis of ALM Tree

After giving the definition of stability of application layer multicast tree, we now analysis the typical topology model of ALM tree and its stability model.

4.1 Multicast Tree Topology Model

Before analyzing the stability of an ALM tree, we first introduce a tree topology model. Without loss of generality, we consider a multicast tree as composed of three parts of links, as shown in Fig.2.

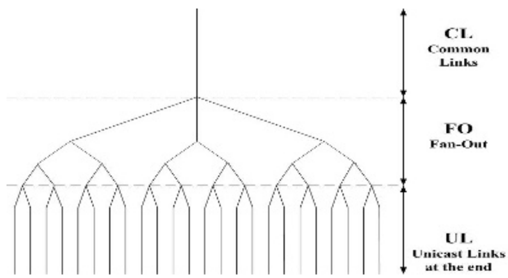

Fig. 2. A model for application layer multicast tree

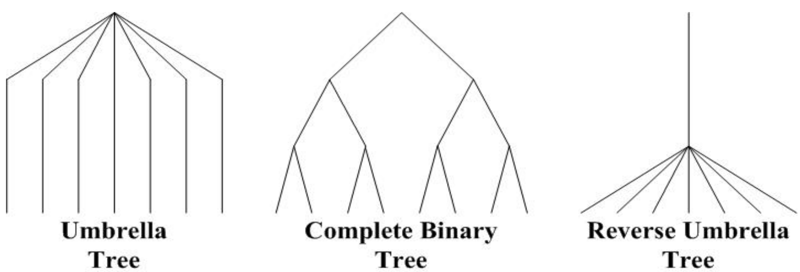

Fig. 3. Three basic tree types

(1) Common links: height is CL. That is for all receiving hosts.

(2) Fan shape links: height is FO. The degree of the uppermost nodes in this fan shape link is *k*, while the degree of other nodes is 2.

(3) Line shape links: height is UL. In these links, the degree of every node is 1, and there is no common links between nodes.

Given different combination of three types of links, we will have different types of multicast trees. We will focus on three representative basic types, as shown in Fig. 3.

(1) Umbrella tree: The leftmost tree type in Fig.3, which begins with a short sector link (FO = 1), followed by a long line style link (very large UL value). The unique feature of the umbellate form multicast tree is that few links are shared among receiving hosts. Such tree types are similar to the Mbone.

(2) Complete binary tree: The tree type in the middle of Fig.3, where every node has two child nodes (very large FO value).

(3) Reverse umbrella tree: The rightmost tree type in Fig.3, where the subgroup first transmits along a long general purpose link, then ends in a short sector link (fairly large CL, FO=1).

4.2 The Stability Models of the Basic Tree Types

We now give the stability factors of the three basic tree types. For simplicity, we consider only homogeneous trees, i.e., for $\forall i, j$, we have $0 < p_{ij} = p < 1$. Then the following theorem holds.

Theorem 2: For a relevance-marking application layer multicast tree T (height $=m$) defined in definition 2, if we use $\Psi_d(P_{ij}, T)$ to represent the probability for node P_{ij} to become a bottleneck node, we have

$$\Psi_d\left(P_{ij}, T\right) = p(1-p)^{(i-1)}, \text{if } 1 \le i \le m \ . \tag{3}$$

4.2.1 The Stability Model of Umbrella Tree

Theorem 3: For an umbrella tree T (height=m, source node grade=k), we have

Conclusion 3.1: the number of nodes $N(T)$ in a multicast tree can be expressed as:

$$N(T) = m \cdot k + 1 \ . \tag{4}$$

Conclusion 3.2: the average change of node number $E(\Delta(T))$ caused by bottleneck nodes in a multicast tree can be expressed as:

$$E(\Delta(T)) = k \cdot \sum_{i=1}^{m} \Psi_d\left(P_{ij}, T\right)(m-i+1) = k \cdot \sum_{i=1}^{m} p(1-p)^{(i-1)}(m-i+1) \ . \tag{5}$$

Conclusion 3.3: the stability factor $S(T)$ in a multicast tree can be expressed as :

$$S(T) = 1 - E(\Delta(T))/N(T) = 1 - \frac{k}{m \cdot k + 1} \cdot \sum_{i=1}^{m} p(1-p)^{(i-1)}(m-i+1) \ . \tag{6}$$

4.2.2 The Stability Model of 2-Ary Balanced Tree

Theorem 4: For a 2-ary balanced tree defined in *definition 2* (height $= m$), we have:

$$N(T) = 2^{m+1} - 1 \ . \tag{7}$$

$$E(\Delta(T)) = \sum_{i=1}^{m} 2^i \cdot \Psi_d\left(P_{ij}, T\right)(2^{m-i+1} - 1) = \sum_{i=1}^{m} p(1-p)^{(i-1)}(2^{(m+1)} - 2^i) \ . \tag{8}$$

$$S(T) = 1 - E(\Delta(T))/N(T) = 1 - \frac{1}{2^{m+1} - 1} \cdot \sum_{i=1}^{m} p(1-p)^{(i-1)}(2^{(m+1)} - 2^i) \ . \tag{9}$$

4.2.3 The Stability Model of Reverse Umbrella Tree

Theorem 5: For a reverse umbrella tree T (height $= m$, source node grade= k) defined in definition 2, if $CL = m-1$, $FO = 1$, the dimension number of chain circuit FO is k, then we have:

$$N(T) = m + k \ . \tag{10}$$

$$E(\Delta(T)) = k \cdot p(1-p)^{(m-1)} + \sum_{i=1}^{m-1} p(1-p)^{(i-1)}(m-i+k)\ . \tag{11}$$

$$S(T) = 1 - \frac{k}{m+k} \cdot p(1-p)^{(m-1)} - \frac{1}{m+k} \cdot \sum_{i=1}^{m} p(1-p)^{(i-1)}(m-i+k)\ . \tag{12}$$

Proof can be found in [11].

4.3 Numerical Results

We now use Matlab to observe the stability of these three basic types of multicast tree. We use the following default settings: the height of the multicast tree is 7, i.e., $CL + FO + UL \le 7$; the maximum node number of the multicast tree is 128. We focus on the results describing the variation of multicast tree factors compared to node numbers, the conclusion is shown as in Fig.4.

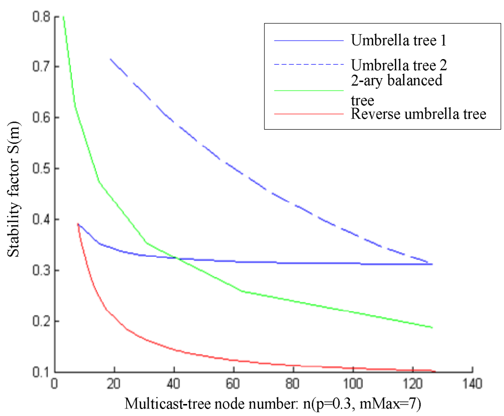

Fig. 4. The variation of stability factors

For an umbrella tree, we investigate the impact of an increase of nodes on the stability of the tree is in two aspects: (1) depth-first: first, we generate an umbrella tree whose height is 7 and dimensional number 1, node number 8; then we increase the dimensional number; the node number increases by 7 when the dimensional number is increased by 1; stop increasing the dimensional number when it reaches 18,so the node number is 7*18+1=127. (2) breadth-first: first generate an umbrella tree whose height is 1 and dimensional number 18; gradually increase the height of the tree, the node number increasing by 18 each time; stop increasing the height of the tree when it reaches 7. The node number at this time is still 7*18+1□127.

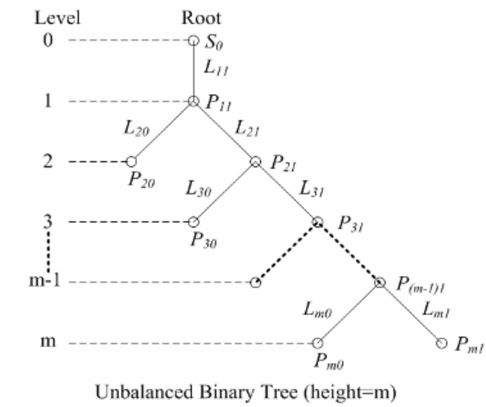

Fig. 5. Unbalanced binary tree (height=m)

For a 2-ary balanced tree, first, we observe a 2-ary balanced tree whose height is 1 and node number 3; then we increase the height of the complete binary tree, 1 by each time; when the height of the tree reaches 6, the node number of the tree is 127.

For a reverse umbrella tree, we generate a tree whose height is 7, and increase the node number by increasing the dimensional number of the sectional nodes. We can see the curve of variation of the stability factors in a multicast tree when the node number increased from 8 to 127 in a reverse umbrella tree.

Several important conclusions can be drawn from Fig.4. As is shown in Fig.4, when having the same numbers, umbrella trees generated through breadth-first method are far more stable than those through depth-first methods. Hence, one important conclusion can be drawn: when there is a new node joining the multicast tree, we should choose a node that has the least height to be the parent node.

Next we will investigate the difference of multicast stability of the three basic tree types when they have the same user nodes. As is shown in Fig.4, among the three types of multicast trees, the umbrella tree is the most stable while the reverse umbrella tree the least. Hence we have the conclusion: nodes of higher competence in an application layer multicast tree should locate at the top of the multicast tree to minimize the value of CL so that the multicast tree could be even more stable.

5 Impact of Stable Nodes

In this section we will discuss the impact of stable nodes (which have zero probability of departure) on the stability of ALM trees. We have researched on two types: unbalanced binary tree and *k*-ary balanced tree. Due to space limit, we only presented results of unbalanced binary tree here. Interested readers can find more results in [11].

5.1 Unbalanced Binary Tree

Theorem 6: For an unbalanced binary tree (height = m) defined in definition 2, as is shown in Fig.5, we have the following conclusions.

Conclusion 6.1: if $m < \infty$, and for arbitrary i and j, $0 < p_{ij} = p < 1$, then the probability for node P_{k1} to become a bottleneck node can be expressed as:

$$\Psi_d(P_{k1}, p, m) = p(1-p)^{(k-1)}, \text{if } 1 \le k \le m-1 \ . \tag{13}$$

The impact of intermediate node P_{k1}'s becoming a bottleneck node on the number of nodes $\Delta(P_{k1}, p, m)$ can be expressed as:

$$\Delta(P_{k1}, p, m) = 2(m-k)+1+(k-1)p, \text{if } 1 \le k \le m-1 \ . \tag{14}$$

The probability that there is no bottleneck node can be expressed as:

$$\Psi(m) = (1-p)^{(m-1)} \ . \tag{15}$$

When there is no bottleneck node in a multicast tree, the change in node number $\Delta(p, m)$ can be expressed as:

$$\Delta(p, m) = m \cdot p \ . \tag{16}$$

The average change of node number $E(p, m)$ in a multicast tree can be expressed as:

$$E(p, m) = p \cdot (2m-1) + (1-p)(E(p, m-1) + p) \ . \tag{17}$$

The stability factor of a multicast tree $S(p, m)$ can be expressed as:

$$S(p, m) = \frac{(2-3p+p^2)}{p(2m-1)} + \frac{2(m-1)}{p(2m-1)} \cdot (1-p)^{m-1} + \frac{2}{2m-1} \cdot (1-p)^m \ . \tag{18}$$

Conclusion 6.2: if $m < \infty$ and for arbitrary i, $p_{i1} = \begin{cases} p, 0 < p < 1, if\ i \ne j \\ 0, if\ i = j \end{cases}$,

The probability for intermediate node P_{k1} to become a bottleneck node $\Psi_d(P_{k1}, p, m, j)$ can be expressed as:

$$\Psi_d(P_{k1}, p, m, j) = \begin{cases} p(1-p)^{(k-1)}, \text{if } 1 \le k < j; \\ 0, \text{if k = j} \\ p(1-p)^{(k-2)}, \text{if } j < k \le m-1 \end{cases} \ . \tag{19}$$

The impact of intermediate node P_{k1}'s becoming a bottleneck node on the number of nodes can be expressed as:

$$\Delta(P_{k1}, p, m, j) = 2(m-k)+1+(k-1)p, \text{if } 1 \le k \le m-1; \ . \tag{20}$$

The probability that there is no bottleneck node $\Psi(m, j)$ can be expressed as:

$$\Psi(m, j) = (1-p)^{(m-2)} \quad . \tag{21}$$

When there is no bottleneck node in the multicast tree, the change of node number in the tree $\Delta(p, m, j)$ can be expressed as:

$$\Delta(p, m, j) = m \cdot p \quad . \tag{22}$$

The stability factor $S(p, m, j)$ in the multicast tree can be expressed as:

$$S(p, m, j) = 1 - \frac{1}{2m-1} \cdot \left\{ \Psi(m, j) \cdot \Delta(p, m, j) + \sum_{k=1}^{m-1} \Psi_d(P_{k1}, p, m, j) \cdot \Delta(P_{k1}, p, m, j) \right\} \cdot \tag{23}$$

Proof can be found in [11].

5.2 Numerical Results

We use Matlab to investigate the impact of different factors on multicast stability. First, we consider the impact of different parameters on the bottleneck nodes, and then we study the impact of these parameters on the stability of the multicast tree.

5.2.1 Impact of User Departure

We will focus on an unbalanced binary tree. Fig.6 (a) presents the results of the variation of stability factors against the node departure probability p in 10 unbalanced binary trees whose height varies from 1 to 10. We can see in the figure that as p increases the stability of the multicast tree decreases rapidly; the decreasing rate of S is in proportion to the height m of the multicast tree. For an unbalanced binary tree whose height is 10, even if the node departure probability p is only 0.1, there will be 40% nodes departing from the multicast tree (multicast tree stability factor is 0.6).

5.2.2 Impact of the Height of a Stable Node on the Stability Factor

From conclusion 6.2, we can see the change of the stability factors in an unbalanced binary tree when stable nodes appear. In Fig.6 (b), suppose the height of an unbalanced binary tree m=6, the height j of stable nodes varies from 0 to 5, i.e., the departing probability of user node P_{j1} is $P_{j1=0}$. Specifically, j=0 means that there is no stable node in the multicast tree. Just as we expect, the existence of stable nodes will improve the stability factors in a multicast tree. Especially, with the increase of the departing probability p of unstable nodes, the promotion of stability is more apparent. It is worth noting is that when j=1, i.e., when the user node on the uppermost level is stable, the stability of the multicast tree increase considerably.

5.2.3 Impact of the Height of a Multicast Tree on the Stability Factor

Fig.6 (c) presents the curve of the change of stability factor S against the height m in an unbalanced binary tree. The curves in the figure represent respectively the locations of stable nodes in the multicast tree. We can see very clearly that as the height of the tree increases, the stability of the tree drop rapidly. But the existence of stable nodes promotes the stability of the multicast tree. Especially, when the height

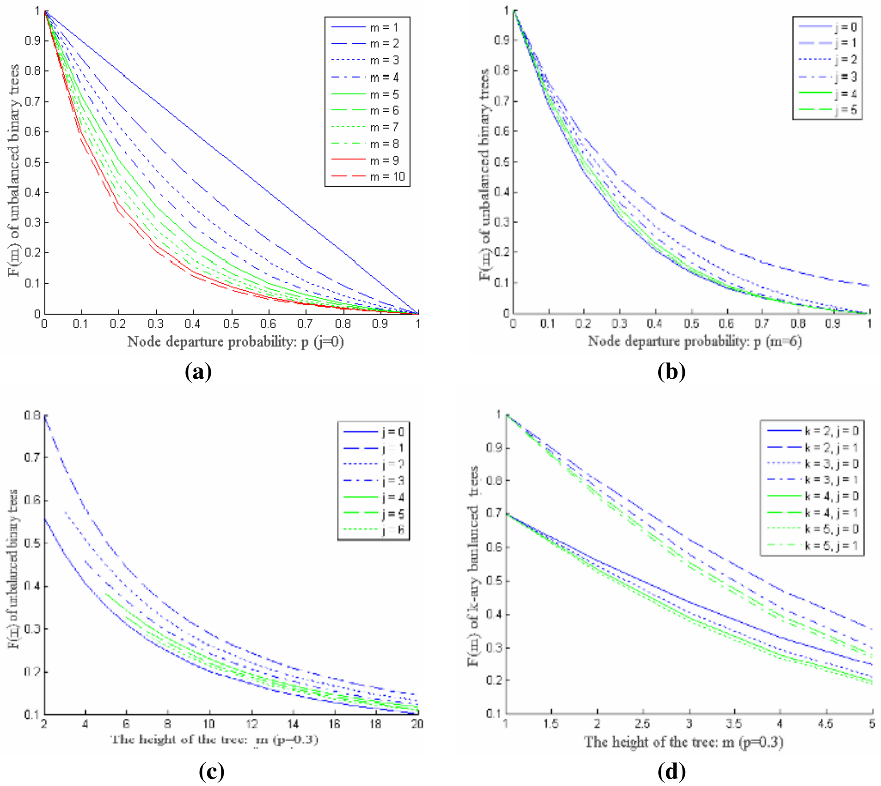

Fig. 6. Numerical Results

of the tree is not large (e.g. $m = 4$), the stable nodes can even improve the stability factor from 0.4 to 0.6.

We continue to observe the impact of the height of the tree on the stability factor. In Fig.6 (d) we make a comparison of multicast trees of different dimensions and different stable nodes to see their impact on the stability factor. We can see in the figure, for multicast trees with the same height, the larger the dimensional number k, the less stable the tree is. However, this impact is not as apparent as is shown in the figure. But when a stable node appears at the nearest node to the source ($j = 1$), the stability of the multicast tree is greatly promoted.

6 Conclusion

In this paper, we presented an initiative discuss on the impact of dynamic user behaviors to the stability of ALM trees. We proposed a novel stochastic model that captures the (in)stability characteristics of an ALM tree. Our model has considered various important factors related to the tree stability, and we have derived closed-form solutions to a class of typical multicast trees. These solutions have established a theoretical basis for designing stable ALM tress. From the results of the model, we suggest the following enhancements to improve the stability:(1) to include high performance and stable nodes, and place them on the upper level of the multicast tree; (2) to reduce the height of the multicast tree as much as possible, and choose parent nodes that have the smallest height when the child nodes join the multicast tree.

Acknowledgement

This work was supported by NSFC (No 60473082). We thank Dr. SHI Feng for developing initial model of ALM tree.

References

1. Peercast Website: http://www.peercast.org/
2. X. Zhang, J.C. Liu, Bo Li, and T.-S. P. Yum: Cool Streaming/DONet: A Data-Driven Overlay Network for Efficient Live Media Streaming. INFOCOM (2005)
3. Y.-H. Chu, S. G. Rao, and H. Zhang: A Case for End System Multicast. ACM Sigmetrics (2000)
4. Y.-H, Chu, S.. G. Rao, S. Seshan, and H. Zhang: Enabling Conferencing Applications on the Internet using an Overlay Multicast Architecture. ACM SIGCOMM (2001)
5. P. Francis. Yoid: Extending the Internet Multicast Architecture. White Paper http://www.icir.org/yoid
6. J. Jannotti, D. K. Gifford, K. L. Johnson, M. F. Kaashoek and J. W. O'Toole, Jr: Overcast: Reliable Multicast with an Overlay Network. OSDI (2000)
7. F. Shi, J.P. Wu, and K. Xu. Impact of congestion on the stability of a multicast tree in cumulative layered multicast. IEE Proceeding Communication, (2003) 150(5):371-376
8. P.V. Mieghem, M. Janic, Stability of a Multicast Tree. INFOCOM(2002)
9. X. Zhang, K.G. Shin, Statistical Analysis of Feedback-Synchronization Signaling Delay for Multicast Flow Control. INFOCOM(2001)
10. F. Shi, J. Wu, K. Xu, Stability of a Multicast Tree in Cumulative Layered Multicast Congestion Control. IPCCC(2003)
11. K XU, JC Liu, F. Shi. On the Stability of Application-Layer Multicast Tree (extended version). http://netlab.cs.tsinghua.edu.cn/~xuke/ALMStability 20051127.pdf