Performance and Incentive of Teamwork-based Channel Allocation in Spectrum Access Networks

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***Abstract*—Recent years have witnessed the great popularity of dynamic spectrum access networks. Such an approach is adopted between three players: government, Internet Service Providers (ISPs) and end-users. ISPs need to purchase spectrum from the government before subletting it to end-users, but currently most researches focus on the subletting process and ignore the purchasing process.**

In this paper, we try to investigate the game between government and ISPs in spectrum access networks. In this framework, the former aims to optimize user experience yet the later want to maximize their own profits. Such a conflict of interests introduces significant challenges to ensure end-user's performance and thus leads to a severe bottleneck to the spectrum access networks. Inspired by cooperative trends among users, we proposed a novel Channel Allocation model based on Teamwork (CAT). This approach considers both ISP's respective bands and end-user's experience and enables a smart profit sharing algorithm to address the problem. The evaluation results indicate that CAT improves the overall social welfare by about 30% than the Vickrey Clarke Groves (VCG) mechanism and obtains higher stability.

***Keywords*—Channel allocation, Spectrum access network, Team-work based, Game theory.**

I. Introduction

SPECTRUM access networks have recently attracted a substantial amount of attentions from both academia and industry. Most researches focus on the game between ISPs and end-users, such as [1], [2]. However, without considering the relationship between governments and ISPs, the optimization of spectrum scheduling will become rather blur. Such questions as how the spectrum is allocated to ISPs, play important roles in the development of spectrum access networks.

To the best of our knowledge, we are the first to explore the gaming between government and ISPs in dynamic spectrum access networks. Our investigations show that the consideration of government introduces new challenges to system analysis. Such as the large communication range and demand-indivisibility issues. To be specific, in traditional spectrum access issues, the buyers are end-users, whose communication ranges are small, so the interference between them is unlikely to happen. But when the buyers are ISPs, whose communication range is considerably wide, the allocation process requires well coordination in order to avoid large-scale interference. As to the demand-indivisibility, due to channel heterogeneity, different channel bands are known to have distinguishing functions. When a channel is allocated to an ISP, the spectrum owners cannot sell it again to other ISPs. This is different from the traditional spectrum access networks when different end-users can dynamically share a channel with minimized interference [3], [4].

To address these problems, we carefully examine the relationship between government and ISPs. Specifically, in the rest of this paper, we use primary user (PU) stands for the government, and secondary user (SU) stands for the ISP.[1] Our investigations indicate that the SUs have a clear trend to cooperate. We therefore designed a Channel Allocation model based on Teamwork (CAT). In CAT, PU has heterogenous spectrum to sell and SUs need the spectrum to further provide their services to the end-users. We then give the comparative experiment between CAT and Vickrey Clarke Groves (VCG) mechanism [5]. Experiment results show that CAT model obtains 80% to 95% of the optimal social welfare, while VCG mechanism gets only 60% to 80% on average.

The rest of this paper is organized as follows. Section II reviews the related work. We give the design goal of this paper in Section III. Main system framework

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[1] In the traditional game between ISPs and end-users, ISPs act as PUs and end-users act as SUs, but in the game between ISPs and the government, ISPs act as buyers and government acts as sellers, so we use PU stands for the government, and SU stands for ISP.

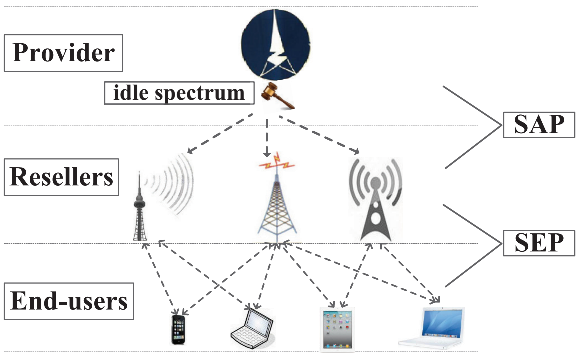

Fig. 1: Three components in Spectrum Area.

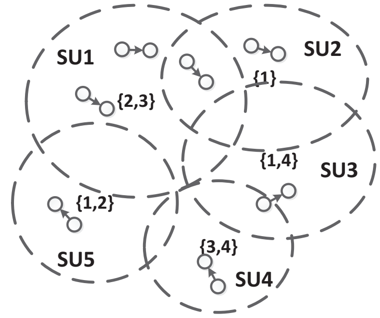

Fig. 2: Collision domain with 5 SUs and each of them has one or more channel demands. The dashed circle denotes the communication range of an SU, and the two small circles connected with an arrow denote the communicating parties.

model CAT is shown in Section IV. Section V shows the simulation results. Finally, Section VI concludes this paper and points out the future work.

II. Related Work

As ordinary users, we use channels nearly every day, but how do these bands be assigned to us? In order to clarify this question, we need to understand the following two processes: One is how does the government sell spectrum to ISPs. The other is how do ISPs provide accessible channels to end-users, as shown in Fig.1. In fact, the government acts as "provider", and ISPs act as "resellers". The two processes are referred as the sale process (SAP) and the service process (SEP).

In SAP, providers would sell the available idle spectrum, which is state-owned resource in most countries to resellers. One of the largest auctions in history is the 2000-2001 European auctions of third generation (3G) mobile telecommunication (or UMTS) licenses [6]. In SEP, resellers provide various services to end-users, such as wireless mesh network service [7] [8], multicast and broadcast service [9]. Additionally, end users can dynamically access idle spectrum without affecting the use of authorized users, such as the secondary TV white spaces network [2] introduced in [1], the "Super WiFi", future home area networks and smart metering in [10].

Recently, game theory has become a common approach to solve the wireless network problems. Cooperative games have been widely used to solve the problems in wireless networks [11]. In [12], the authors examined the issue of adaptive-width channel allocation and guaranteed convergence to a dominant strategy equilibrium by proposing a charging scheme. To take individual rationality into consideration, many researches adopt non-cooperative games. For instance, Feng *et al.* proposed a truthful double auction scheme in [13] and took spectrum difference in space and frequency into account. In their opinions, providers have different spectrum and communication ranges, so buyers are allowed to express their personalized preferences explicitly.

To conclude, many researchers are committed to optimize the SEP process. While in reality, without SAP, SEP will become rather blur. The British economist Paul Klemperer, who analyzed British 3G telecom licenses gave the conclusion in [14] that auction design is not "one size fits all". Therefore, the analysis of SAP is in urgent need.

III. Design Goal

In this paper, we focus on the process that government (PU) sells channels to ISPs (SUs), and aim at maximizing social welfare by designing a spectrum allocation scheme in which selfish SUs would form legal teams to get higher spectrum utility. The key problem is the selling method in SAP, so we leave out the situation that SUs resell the channels to end-users.

A. The interference model

We adopt the commonly-used model in most channel allocation problems. In this model, every SU_i has his communication range r_i with a radius of l_i, and all nodes (end-users) of an SU are in this area. And node n will be interfered by node v if n shares the same band with v in the overlapping area. We assume that the packets in all communication pairs are backlogged [12] [15], which means that every pair has infinite packets to send. As to the communication range, we assume that r_i and r_j will interfere with each other if they have the overlapping area and use the same channel. As shown in Fig. 2, SU_2 and SU_3 cannot use Channel 1 simultaneously, but SU_2 and SU_5 can.

B. Design goals

We assume there are m channels and n SUs. The problem is that when aiming at maximizing social welfare, whom should the PU sells the channels to. There are several challenges here. First, each SU has

[2]The freed TV spectrum in the VHF/UHF band from 54MHz to 698MHz are usually called TV "white spaces".

TABLE I: Variable definitions.

Variable	*Meaning*
W_i	The welfare SU_i can make to society
WTP_i	The highest willingness to pay of SU_i
P_i	The final profit of SU_i
δ_i	The dividend of SU_i from his team
B_i	The highest bid value of SU_i
S_i^j	The Shapley value of SU_i in $team_j$
$\tilde{P}_i$	The final profit of $team_i$
$\tilde{B}_i$	The bid value of $team_i$

his own communication range that cannot be interfered. Second, all requirements are non-replaceable due to spectrum heterogeneity. Third, in SAP, it is a complex MC problem instead of a simple SC one.

To solve this complex problem, we model it to a maximization problem. The ultimate aim is to maximize the total social welfare. Giving some variable definitions in Table I, we model the problem as follow:

$$MAX \sum_{1\le i\le n} W_i \tag{1}$$

$$s.t. \begin{cases} W_i = B_i + P_i, & if\ SU_i\ wins \\ W_i = 0, & if\ SU_i\ loses \\ B_i \le WTP_i + \delta_i & \\ \delta_i = f_1((S_i^j), \tilde{P}_i), & SU_i \in team_j \\ \tilde{P}_i = f_2(WTP_i, \tilde{B}_i), & SU_i \in team_j \\ \tilde{B}_i = f_3(Min(WTP_i, \delta_i)) & \\ W_i > 0 & \\ P_i > 0 & \end{cases} \tag{2}$$

Where “wins” means SU obtains what he needs, “loses” means he obtains nothing. The first equation is valid because the value he can create for the society is consist of two part, one is the cost (B_i), the other is the profit (P_i). But if the SU hasn’t got any channels, the value he can create is 0. The third equation denotes that the highest bid of an SU is depended on his WTP and the dividend from his team. This dividend is calculated by a distribution method in game theory (denoted as function $f1$). The team profit is calculated by WTPs and team bid value (denoted as function $f2$), while team bid value is depended on the minimum bidding ability of members (denoted as function $f3$). We will further explain these constraints later in our models.

IV. Channel Allocation based on Teamwork - CAT

CAT is modeled on the base of some knowledge in economics, such as the Nash bargaining solution and the Shapley value, and CAT also takes collective and individual rationality into account.

With the above knowledge, we now consider the scenario that PU has some idle channels to sell and every SU needs one or multiple. In the primary model, we leave out the condition of multi-collision domains, and we set the prices of heterogeneous bands as the same. We assume each SU has a powerful radio that can operate over the entire range of the managed spectrum. This means one spectrum band can only be sold to one SU, as collisions will occur in the same bands of different SUs.

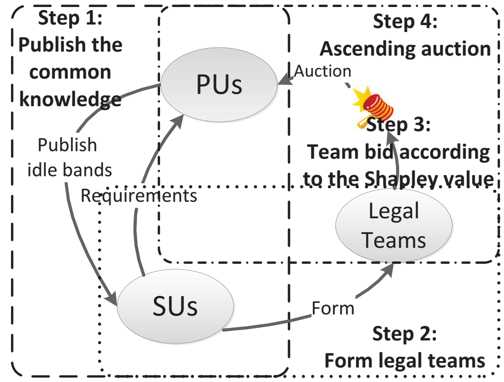

Fig. 3: The flow sheet of CAT with 4 steps.

We assume that if an SU has a demand quantity of n_1 channels and his profit is m_1 dollars per channel, his total payoff $payoff_{full}$ is $n_1 \times m_1$ if he gets all the n_1 channels. If he just gets n_2 channels ($n_2 < n_1$), his profit will be cut into $\frac{n_2}{n_1} \times m_1$ dollars per channel [3], and his total payoff ρ_{cut} will be $n_2 \times \frac{n_2}{n_1} \times m_1$. Therefore,

$$\rho_{cut} < \rho_{full} \tag{3}$$

As a selfish player, every SU will be eager to get all the desirable channels to maximize his payoff. So we try to design a model, which considers one’s requirements as a whole. Aiming at gaining higher social welfare, we design CAT, which can be summarized as: PUs publish idle bands; SUs report their demands; SUs form legal teams and ascending auction. Later we will improve this primary model to M-CAT to adapt to realistic environment.

The flow sheet of our models is shown as Fig. 3. The implementation of the CAT model is described as follows:

A. Step1: Publishing the common knowledge

Firstly, PU should publish the idle spectrum bands, i.e., $band_1$ to $band_m$, and information about these m available bands is common knowledge among all SUs. We assume there are a set of SUs in the game $\mathbf{P} = \{SU_1, SU_2, \cdots, SU_n\}$, and each SU has a non-empty requirement set N_i. Then PU publishes the threshold β of spectrum utilization and the unit price p of every spectrum band.

In order to ensure the spectrum efficiency, PU uses β as the minimum selling threshold, that is, at least $m \times \beta$ channels should be sold out in this process.

With the above knowledge, all SUs report their demands N_i simultaneously, and each N_i contains SU_i’s band requirement:

$$N_i = \{band_j, \cdots\},\ 1 \le i \le n,\ 1 \le j \le m$$

With the variety of requirements, SUs could form legal teams without conflicts.

[3] Here we use a linear special case, while it can be replaced by other functions and gets the same results.

Algorithm 1 LegalTeamCalculation()

```
for i = 1 : 2^n − 1 do
    initialization();
    lp = next loop
    for a = 1 to n do
        for b = 1 to m do
            if SU_needs(a,b) = 1 then
                text(conflict);
            end if
        end for
    end for
    if non-conflicting and test(utility ≥ β) then
        form a legal team;
    end if
end for
```

Fig. 4: Legal Team Calculation Algorithm

B. Step2: Forming legal teams

As there is a threshold β in the system, one SU cannot purchase $m \times \beta$ bands by himself. Since every SU has individual rationality, they would like to be in a team. So the SUs with different demands would automatically form teams to achieve the sale threshold. Only these alliances (i.e., "legal teams" in our subsequent paper) have chances to bid spectrum bands later, and SUs in one team would bid as a union.

We design Algorithm 1 to calculate all the legal teams. Algorithm 1 takes exponential time $O(2^n)$ to calculate the teams. After that, we can obtain all the legal teams. In Step 3, we will show how the members work together to raise their bids and get the bands.

C. Step3: Team bidding according to Shapley value

In this step, we introduce the residual profit distribution mechanism, through which a team can raise the ability to win. We choose Shapley value which was proposed in 1953 to solve the profit sharing problem. In our distribution method, we define the profit assigned to an SU is in direct proportion to the contribution he made to his team.

Now we discuss how to calculate the profit of a team made up by n SUs, denoted as SU_1 to SU_n. Let R_i^j $(1 \leq i \leq n)$ denote the spectrum utilization rate at SU_i in $team_j$. Given the threshold β, we obtain the requirement of a legal team that

$$\sum_{i=1}^{n} R_i^j \geq \beta \tag{4}$$

Thus, we define the profitability η_j of a team as Equation 5. For a legal $team_j$, the ability is $\sum_{i=1}^{n} R_i^j$, while the illegal team's ability is the square of utility.

$$\eta_j = \begin{cases} \sum_{i=1}^{n} R_i^j, & \text{if } \sum_{i=1}^{n} R_i^j \geq \beta \\ (\sum_{i=1}^{n} R_i^j)^2, & otherwise \end{cases} \tag{5}$$

Now we consider the residual profit of a team. Let WTP_i denote SU_i's highest WTP, $\tilde{B}_j$ denote the total price offered by legal $team_j$, and A_j represents the total number of required bands in $team_j$. Then the bid value of $team_j$ made up by SU_i ($1 \leq i \leq n$) should be calculated as follow:

$$\tilde{B}_j = min\{WTP_i + \delta_i\} \times A_j, \ s.t. 1 \leq i \leq n \tag{6}$$

As the total residual profit is $\sum WTP_i - \tilde{B}_j$, the members in one team can share the team profit. The δ_i in Equation 6 is the dividend for SU_i in $team_j$, and this value depends on the contribution he made to his team, which is related to the Shapley value.

Before giving the calculation of Shapley value, we introduce the marginal contribution. υ denotes the monetary benefits generated by a coalition. For example, we denote $\upsilon(C)$ as the profit produced by Coalition C. When SU_i joins the coalition, the total benefits raises. Thus, the marginal contribution of SU_i is defined by

$$\Delta_{SU_i}(\upsilon, C) = \upsilon(\{SU_i\} \bigcup C) - \upsilon(C) \tag{7}$$

The Shapley value φ is defined by

$$\varphi_i(\upsilon, team) = \frac{1}{N!} \sum_{\tau \in \Pi} \Delta_i(\upsilon, C(\tau, i)), \forall i \in C \tag{8}$$

where N is the number of the coalition $team$, Π denotes all the $N!$ different orderings of C, and $C(\tau, i)$ is the set of SUs preceding i in the ordering τ. Hence, Equation 8 is the expected marginal contribution SU_i made to the set of SUs preceding in all orderings.

We designed Algorithm 2 to compute Shapley value. Though the complexity of algorithm 2 is $n!$, there will not be too many ISPs in any country, the algorithm complexity is acceptable.

So far, we have distributed the residual profit of a team according to these Shapley values.

Next, we analyze the δ in Equation 6, which is the dividend from team. The value of δ is depended on individual's Shapley value and the total residual profit of his team. Now we present the calculation process as:

$$\delta_i = B_i - WTP_i = (\sum_{SU_k \in team_j} WTP_k - \tilde{B}_j) \times S_i^j \tag{9}$$

Algorithm 2 ShapleyCalculation()

```
for i = 1 to n do
    RATE = 0;
    for j = 1 to n do
        if RATE < β and RATE + R_j ≥ β then
            S_j = S_j + RATE + R_j − RATE^2;
            RATE = RATE + R_j;
        else
            if RATE < β and RATE + R_j < β then
                RATE = RATE + R_j;
                S_j = S_j + RATE^2 − (RATE − R_j)^2;
            else
                if RATE ≥ β then
                    RATE = RATE + R_j;
                    S_j = S_j + RATE;
                end if
            end if
        end if
    end for
end for
S = S/RATE/num_SU!
```

Fig. 5: The Shapley Value Calculation Algorithm

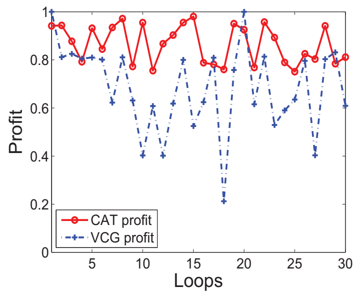

Fig. 6: The total profit of 5 channel bands and 8 SUs in CAT.

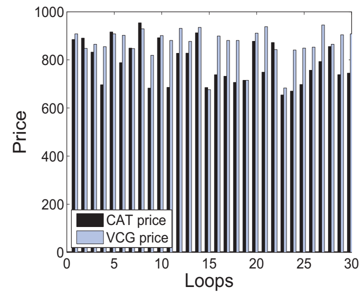

Fig. 7: The selling price of 5 channel bands and 8 SUs in CAT.

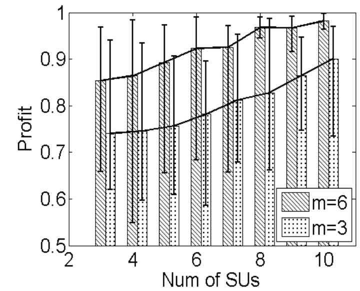

Fig. 8: The impact of the number of SUs.

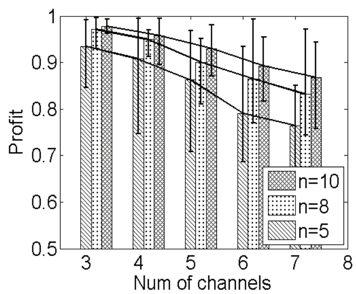

Fig. 9: The impact of the number of channels.

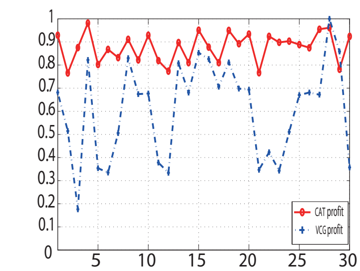

Fig. 10: The total profit of 6 channel bands and 10 SUs in CAT.

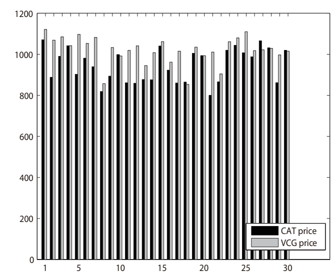

Fig. 11: The selling price of 6 channel bands and 10 SUs in CAT.

where B_i is the highest price SU_i could bid. The difference between B_i and WTP_i is the δ_i in Equation 6, which is the team surplus distributed to him according to his Shapley value. Moreover, the calculation method of $\tilde{B}_j$ is in Equation 6. Combining Equation 6 and Equation 9, we get B_i, then $\delta_i = B_i - WTP_i$.

Now we have the bid value of SUs in all teams. Note that this bid value may be higher than their WTPs. With these values, we can get the highest team price, which depends on the member's WTPs. That is, $\tilde{B}_i = min\{WTP_j + \delta_j\} \times A_i$. At this point, the auction peocess among legal teams can start.

D. Step4: Ascending auction

Now all of the legal teams are prepared to begin the auction. We adopt the ascending auction (British auction) here and use p as the base price, then all legal teams could make a markup price if all their members agree. We set the markup to θ, that is, if the market price is p' and $team_i$ wants to bid, the bid value should be at least $p' + \theta$, and the price of each band is $\frac{p'+\theta}{A_i}$. So the revenue function of each SU is:

$$P_i = W_i - \frac{p' + \theta}{A_i} * n(N_i) \geq W_i - (WTP_i + \delta_i) \quad (10)$$

$n(N_i)$ is the number of required bands, $\frac{p'+\theta}{A_i} * n(N_i)$ should be less than $WTP_i + \delta_i$, otherwise, this SU will exit this auction. Consequently, his team will fail, too. The dynamic game is repeated until only one team remains, which becomes the winner. SUs in this team will get their required spectrum bands.

V. Evaluations

In this section, we first introduce the evaluation setup and explain the experimental parameters, then we give the comparative experiments between CAT and VCG mechanism. VCG mechanism is a relatively mature auction mechanism, which can realize good efficiency in either homogeneous or heterogeneous spectrum auction.

A. System setup

In the simulations, we can set the number of channels and SUs, idle spectrum to be auctioned, the sale threshold β, the base prices of bands and the increasing price θ of each auction. Here we propose the Demand Matrix and WTP vector:

- Demand Matrix. We assume there are m spectrum bands and n SUs, and the Demand Matrix is two-dimensional by $n \times m$, which is randomly generated and non-empty;
- WTP vector. In simulations, we can set the lowest and highest WTPs, and the vector is uniformly distributed in this range, which meets the real world situation.

B. Experimental evaluation

Refering to [4], we assume there are 5 spectrum bands and 8 SUs with WTPs from 150 to 200, and θ is 5. The 30-round simulation results are shown in Figs. 6 and 7.

In Fig. 6, we can conclude that the social profit of CAT is generally higher than that of VCG mechanism.

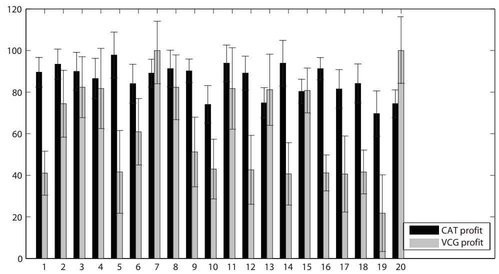

Fig. 12: The fluctuation of social profit in CAT model.

The average is 91% of the optimal social welfare, while VCG mechanism obtains 68%, and our result is relatively stable in different simulations. Fig. 7 shows the final selling price. Although the selling price of CAT is occasionally a little lower than VCG mechanism, the difference is no more than 5% on average.

In order to test the influence on the performance of CAT, we conduct experiments to verify: (i) number of SUs, (ii) number of channels. We choose the scale according to references [6], which described the real auctions in Europe [4]. We plot our results in Figs. 8 and 9, each being an average value of ten experiments. We give the error bar and observe that: (i) when the number of channels is fixed, the more SUs, the higher social profit will be; (ii) when the number of SUs is fixed, an increasement in the number of channels will reduce the social profit. This character still holds in the extended model because when there are more channels, the minimum sales will grow, increasing the difficulty of forming teams, and reducing the total profit. Moreover, it is possible that the results will not change monotonically with variables because the experiment parameters here are not preprogrammed, but generated randomly every time to keep generality.

Further, we specify 6 bands and 10 SUs [12], and conduct several experiments each with 30 randomized trials, as shown in Figs. 10 and 11. For a more general result, we again change the two numbers and obtain similar results.

We repeat this cycle for 20 times, calculate the variance for each cycle, and draw the variance histogram in Fig. 12, from which we can conclude that the profit our model produces is not only higher, but also more stable than VCG mechanism.

VI. Conclusion

In this paper, we for the first time analyzed the whole process of spectrum allocation, including the sale process *SAP* between government and ISPs, and the service process *SEP* between ISPs and end-users. As *SAP* was left out by most researches, we are the first to investigate the gaming between governments and ISPs. To address the new challenges raised by SAP, we designed a Channel Allocation framework based on Teamwork (CAT) using cooperative game, and this approach considers respective bands as well as end-user's experience and enables a smart profit sharing algorithm. The evaluation results showed that CAT can improve the overall social welfare and obtain higher strategy stability than the VCG mechanism.

[4]There were 5 channels and 5 SUs in the UK auction, 5 channels and 6 SUs in the Italian auction, and 9 bidders for 4 channels in the Swiss auction.

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