# Supplementary File of the IEEE IPCCC Submission

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***Abstract*—This file contains supporting materials of the IPCCC'14 submission - "Achieving Bandwidth Guarantees in Multi-Tenant Cloud Networks Using a Dual-Hose Model".**

## I. Proof

In this file, we refer to the submission file as the *main* file.

### A. VM Placement

***Theorem** 1:* For any feasible placement of a new tenant request $P$ denoted by $\langle N_P, B^a_P, B^e_P, S\rangle$, the bandwidth requirement on each link with the dual-hose model is less than or equal to that with the hierarchical hose model.

**Proof:** To quantify the bandwidth required by tenant $P$'s VMs on each physical link with the dual-hose model, we resort to the communication network shown in Fig. 1(a). [1] Given a specific placement for tenant $P$'m VMs, *e.g.*, placing $p_1$ VMs in the subtree under link $l$ and $p_2$ VMs in the rest ($p_1 + p_2 = N_p$), the max-flow of the corresponding communication network denoted by $F^{dual}_{max}$ is exactly the bandwidth requirement on link $l$.

For convenience of explanation, we create an *intermediate* communication network by slightly modifying the current communication network in Fig. 1(a), where the capacity of edges between two inter-tenant nodes is replaced with the source tenant's aggregate inter-tenant guarantee. For example, the capacity of the edge from node $p^{inter}_i$ to node $q^{inter}_o$ is replaced by tenant $P$'s aggregate inter-tenant guarantee (*i.e.*, $B^{inter}_P$). The max-flow of the intermediate communication network $F^{int}_{max}$ is not less than $F^{dual}_{max}$, because the following inequations hold

$$\begin{aligned} min(p_1B^e_P, q_2B^e_Q) \leq p_1B^e_P \leq (p_1+p_2)B^e_P = B^{inter}_P \\ min(q_1B^e_Q, p_2B^e_P) \leq q_1B^e_Q \leq (q_1+q_2)B^e_Q = B^{inter}_Q \end{aligned} \quad (1)$$

Similarly, the bandwidth requirement with the hierarchical hose model can also be captured by a variation of the communication network as shown in Fig. 1(b) [1]. The max-flow of such a network is denoted by $F^{hier}_{max}$. Now we have $F^{int}_{max} \leq F^{hier}_{max}$. The intuition behind this inequality is as follows. The intermediate and the hierarchical communication network now have the same capacity upper bound for flows traversing edges between inter-tenant nodes, *e.g.*, the upper bound for flows between $p^{inter}_i$ to node $q^{inter}_o$ is $B^{inter}_P$. Due to the decoupling of two types of guarantees for each VM in the intermediate communication network, if the inter-tenant guarantees of VMs of tenant $Q$ on the right side become the bottleneck, VMs of tenant $P$ on the left side decreases their inter-tenant traffic to tenant $Q$. However, with the hierarchical hose model in Fig. 1(b), VMs of tenant $P$ on the left side are likely to react to the bottleneck scenario by increasing the traffic to VMs of tenant $P$ on the right side, so as to maintain the same total amount of sending traffic. Therefore, the max-flow of the hierarchical communication network is not less than that of the intermediate communication network. Now we have $F^{dual}_{max} \leq F^{hier}_{max}$. ∎

[1] As the same with the main file, we refer to physical links in the cloud network as "links" and links in the communication network as "edges".

### B. Bandwidth Allocation

***Theorem** 2:* The bandwidth allocation strategy proposed in Section IV in the main file can achieve efficiency and fairness simultaneously.

**Proof:** From Eq. (6) in the main file, the bandwidth allocated to each type of flows on link $l$ is proportional to their required guarantees on link $l$. Therefore, it is straightforward to find that this bandwidth allocation strategy is work-conserving, because spare capacity on each link can be utilized by active flows traversing the link.

Now we focus on illustrating the fairness in bandwidth allocation. For each VM $p$, the aggregate weight for all its *intra-tenant* flows, denoted by $w^{agg}_p$, follows

$$\begin{aligned} w^{agg}_p &= \sum_{q\in Dst^a(p)} w_{p,q} = \sum_{q\in Dst^a(p)} \min(\frac{B^a_{t(p)}}{N^a_p}, \frac{B^a_{t(q)}}{N^a_q}) \\ &\leq \sum_{q\in Dst^a(p)} \frac{B^a_{t(p)}}{N^a_p} = \frac{B^a_{t(p)}}{N^a_p}N^a_p = B^a_{t(p)} \end{aligned} \quad (2)$$

where $Dst^a(p)$ is a set containing all VMs that have intra-tenant connections with VM $p$.

Therefore, the aggregate bandwidth allocated to VM $p$'s intra-tenant flows on link $l$, denoted by $B^{agg}_p$, follows

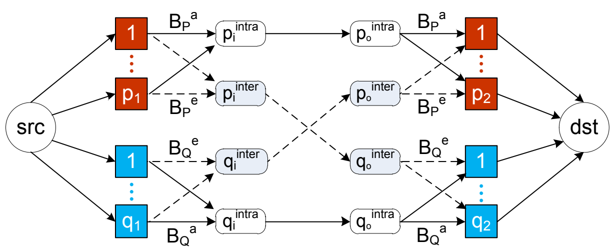
$$B^{agg}_p = \sum_{q\in Dst^a(p)} B_{p,q} = \frac{w^{agg}_p}{W^a_l}C^a_l \leq \frac{B^a_{t(p)}}{W^a_l}C^a_l \quad (3)$$

where the equality holds because of the definition of $B_{p,q}$ in Eq. (7) in the main file and the inequality holds because of the above Eq. (2). Similar illustration is also applied to VM $p$'s inter-tenant flows. Now we have shown the first aspect of fairness, namely, the bandwidth each VM with active flows can acquire should be proportional to its required guarantees.

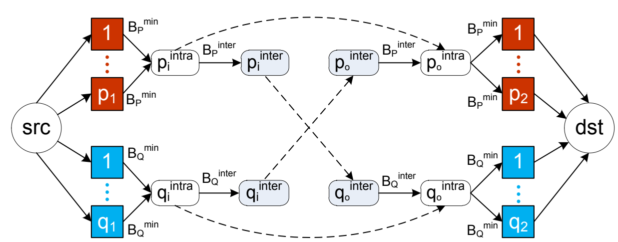
The second aspect of the fairness is provided by Eq. (6) in the main file, where the bandwidth allocated to intra- ($C^a_l$) or inter-tenant flows ($C^e_l$) on any link $l$ is proportional to their respective guarantees. So two types of flows do not interfere with each other in sharing link capacities. ∎

## References

[1] H. Ballani, K. Jang, T. Karagiannis, C. Kim, D. Gunawardena, and G. O'Shea, "Chatty Tenants and the Cloud Network Sharing Problem," in *Proc. USENIX NSDI*, 2013.




(a) Communication network for the dual-hose model




(b) Communication network for the hierarchical hose model [1]

Fig. 1. Communication networks for different models. VMs for different tenants are marked with different colors.